Let α, β, γ, δ, ε represent, respectively, secular aphelion, mean aphelion, mean, mean perihelion, secular perihelion; subscript 0, 2, 3, 5, 6, 7, 8, respectively, Sun, Venus, Earth, Jupiter, Saturn, Uranus, Neptune. Then the harmonic mass-ratios will be represented by the following proport'ns:—

$$m_3 : m_2 :: \gamma_3 : \alpha_2$$
$$m_0 : m_3 :: 24^4 : 1$$
$$m_0 : m_5 :: \varepsilon_5 : \varepsilon_0$$
$$m_5 : m_6 :: \delta_6\gamma_6 : \delta_5\gamma_5$$
$$m_7 : m_3 :: (\beta_7 - \varepsilon_3)\gamma_7 : (\beta_7 + \delta_5)\gamma_3$$
$$m_8 : m_3 :: (\delta_8 - \alpha_3)\gamma_8 : (\delta_8 + \alpha_7)\gamma_3.$$

The accordance between the nodal and computed values is shown in the following table:—

		Nodal.	Computed.	Authority.
Venus	$m_0 \div m_2$	428417	427240	Hill.
Earth	$m_0 \div m_3$	331776	331776	Chase.
Jupiter	$m_0 \div m_5$	1047.872	1047.879	Bessel.
Saturn	$m_0 \div m_6$	3503.22	3501.6	Bessel.
Uranus	$m_0 \div m_7$	22643	22600	Newcomb.
Neptune	$m_0 \div m_8$	19428	19380	Newcomb.

The following points of symmetry and alternation may be noted in the nodal mass-factors of the outer planets:—

1. The tendency to equality of mean orbital *vis viva* in Earth, Uranus and Neptune, as indicated by the factors γ_3, γ_7, and γ_8.

2. The nodal modification of Neptune's mass by Earth's secular aphelion, and of the mass of Uranus by Earth's secular perihelion.

3. The nodal modification of Neptune's mass by its own mean aphelion, and of the mass of Uranus by its mean aphelion.

4. The modification of Uranus by Jupiter, and the corresponding modification of Neptune by Uranus.

NOTE ON DIRECTION.

BY PROFESSOR T. M. BLAKSLEE.

[Ccontinued from page 16.]

WHAT has preceded relative to the point as generatrix and line as path, may assist in giving a clear idea of direction, which is rendered necessary by the increased use of the term in Geometry and Quaternions.

The following additional definitions and propositions are intended to further illustrate the subject.

1. *Def.* The St. surface having the St. line as its G. is called *a plane.*

a. A surface, such that if any two of its points be joined by a St. line the line will lie wholly in the surface, is determined by three of its points; for, revolve it about the St. line joining two of the points till it coincides with the third. If now it be revolved it will no longer contain the third point; ∴ it is determined by three points, or two St. lines, ∴ it is a plane; and since but one plane can be passed through two St. lines, therefore—

b. If any two points of a plane be joined by a St. line, the line will lie wholly in the surface.

2. A regular polygon is one which is both equilateral and equiangular.

3. A *circle* is the path of a point the direction of whose motion is uniformly varied. The circle is evidently the limit of a regular polygon.

4. All the vertices of a regular polygon are equally distant from a point called the centre of the polygon; for, bisect the angles, the resulting triang's are isosceles, and since their bases are equal, the bisectors intersect in a common point.

5. From 3 and 4, the circle may be defined as the path of a point moving so as to remain at a constant distance from a fixed p't called the centre.

6. A line drawn from the centre to any point of the circumference of a circle is called the radius at that point.

7. (As in Roberval's method of tangents) The St. p. of instantaneous motion of the generatrix at any G. of a curved path is called the tangent path at that G.

8. In a circle, the radius is perpendicular to the tangent.

Proof.—Decompose the motion in the direction of the radius and at right angles to it. The component in the direction of the radius must be zero, or the radius would not be constant, as it is by definition.

Prop.—If from any point in one terminus of an angle, and at a distance h from the vertex, a perpendicular, p, be let fall on the other terminus, the distance, b, from vertex to foot of perpendicular, is called the *base*, b and p being the projections of h on the corresponding indefinite St. lines.

Since but one perpendicular can be drawn from a point to a line, a right angled triangle is determined by the hypothenuse and the angle at the base.

If the length of h be denoted by h_1, and the corresponding values of p and b, by p_1 and b_1, then will

$$\frac{p}{h} = \frac{p_1}{h_1} = \text{sine, and } \frac{b}{h} = \frac{b_1}{h_1} = \text{cosine}$$

of angle at base.